Comparing GDE and Conflict-based Diagnosis

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Abstract. Conflict-based diagnosis is a recently proposed method for model-based diagnosis, inspired by consistency-based diagnosis, that incorporates a measure of data conflict, called the diagnostic conflict measure, to rank diagnoses. The probabilistic information that is required to compute the diagnostic conflict measure is represented by means of a Bayesian network. The general diagnostic engine is a classical implementation of consistency-based diagnosis and incorporates a way to rank diagnoses using probabilistic information. Although conflict-based and consistency-based diagnosis are related, the way the general diagnostic engine handles probabilistic information to rank diagnoses is different from the method used in conflict-based diagnosis. In this paper, both methods are compared to each other.

1 INTRODUCTION

In the last two decades, research into model-based diagnostic software has become increasingly important, mainly because the complexity of devices, for which such software can be used, has risen considerably and trouble shooting of faults in such devices has therefore become increasingly difficult. Basically, two types of model-based diagnosis are being distinguished in literature: (*i*) consistency-based diagnosis [2, 8], and (*ii*) abductive diagnosis [7]. In consistency-based diagnosis a diagnosis has to be *consistent* with the modelled system behaviour and observations made on the actual system, whereas in abductive diagnosis the observations have to be *implied* by the modelled system given the diagnosis [1]. In this paper, we focus on consistency-based diagnosis as implemented in the general diagnostic engine, GDE for short, [2]. In addition, particular probabilistic extensions to consistency-based diagnosis as implemented in GDE are considered [2].

There is also a third kind of model-based diagnosis that can be best seen as a translation of consistency-based diagnosis from a mixed logical-probabilistic setting to a purely probabilistic setting, using a statistical measure of information conflict. The method has been called *conflict-based diagnosis*; it exploits Bayesian-network representations for the purpose of model-based diagnosis [4].

Although both GDE and conflict-based diagnosis take consistency-based diagnosis as a foundation, the way uncertainty is handled, as well as the way in which diagnoses are ranked, are different. The aim of this paper is to shed light on the differences and similarities between these two approaches to model-based diagnosis. It is shown that conflict-based diagnosis yields a ranking that, under particular circumstances, is more informative than that obtained by GDE.

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The paper is organised as follows. In Section 2, the necessary basic concepts from model-based diagnosis, including GDE, and the use of Bayesian networks for model-based are reviewed. Next, in Section 3, the basic concepts from conflict-based diagnosis are explained. What can be achieved by the method of probabilistic reasoning in GDE is subsequently compared to the method of conflict-based diagnosis in Section 4. Finally, in Section 5, the paper is rounded off with some conclusions.

2 PRELIMINARIES

2.1 Model-based Diagnosis

In the theory of consistency-based diagnosis [8, 2, 3], the structure and behaviour of a system is represented by a *logical diagnostic system* $\mathcal{S}_L = (\mathrm{SD}, \mathrm{COMPS})$, where

- SD denotes the *system description*, which is a finite set of logical formulae, specifying structure and behaviour;
- COMPS is a finite set of constants, corresponding to the *components* of the system that can be faulty.

The system description consists of *behaviour descriptions* and *connections*. A behavioural description is a formula specifying *normal* and *abnormal* (faulty) functionality of the components. An *abnormality literal* of the form A_c is used to indicate that component c is behaving abnormally. whereas literals of the form $\neg A_c$ are used to indicate that component c is behaving normally. A connection is a formula of the form $i_c \equiv o_{c'}$, where i_c and $o_{c'}$ denote the input and output of components c and c', respectively.

A *logical diagnostic problem* is defined as a pair $\mathcal{P}_L = (\mathcal{S}_L, \mathrm{OBS})$, where $\mathcal{S}_L$ is a logical diagnostic system and OBS is a finite set of logical formulae, representing *observations*.

Adopting the definition from [3], a diagnosis in the theory of consistency-based diagnosis is defined as follows. Let Δ_C consist of the assignment of abnormal behaviour, i.e. A_c, to the set of components $C \subseteq \mathrm{COMPS}$ and normal behaviour, i.e. $\neg A_c$, to the remaining components $\mathrm{COMPS} - C$, then Δ_C is a *consistency-based diagnosis* of the logical diagnostic problem $\mathcal{P}_L$ iff the observations are consistent with both the system description and the diagnosis; formally:

$$\mathrm{SD} \cup \Delta_C \cup \mathrm{OBS} \nvDash \bot.$$

Here, $\nvDash$ stands for the negation of the logical entailment relation $\vDash$, and $\bot$ represents a contradiction.

Usually, one is in particular interested in *subset-minimal* diagnoses, i.e. diagnoses Δ_C, where the set C is subset minimal. Thus, a subset-minimal diagnosis assumes that a subset-minimal number of components are faulty; this often corresponds to the most-likely diagnosis.
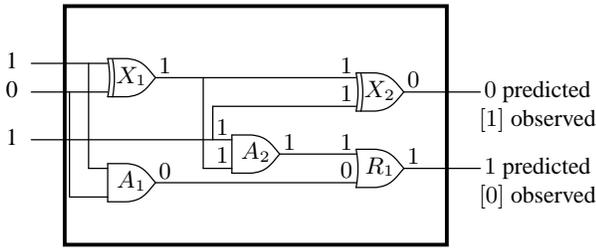
Figure 1. Full adder with all outputs computed under the assumption of normality and observed and predicted outputs; i_1 (1), $\bar{i}_2$ (0) and i_3 (1) indicate the inputs of the circuit and o_1 (1) and $\bar{o}_2$ (0) its observed outputs.

EXAMPLE 1 Figure 1 presents the full-adder example, which consists of two AND gates ($A1$ and $A2$), one OR gate ($R1$) and two exclusive-OR (XOR) gates ($X1$ and $X2$). Note that the *predicted* output $\bar{o}_1$ contradicts with the *observation* o_1, which is also the case for gate $X2$. As a consequence, the assumption that all components are behaving normally is invalid; thus, this is *not* a consistency-based diagnosis. However, a consistency-based diagnosis would be to assume the malfunctioning of component $X1$, as this would restore consistency. □

2.2 GDE

Next, GDE is briefly described, where [2] is used as a point of reference; however, the terminology defined above in this paper is adopted throughout this section. For example, where [2] speaks of a 'candidate' in this paper the term 'diagnosis' is used.

The logical reasoning implemented by GDE can best be seen as an efficient implementation of consistency-based diagnosis. GDE can also deal with uncertainty by attaching a prior probability of malfunctioning to components. After an observation is made, the prior probability becomes a posterior probability, conditioned on this observation. Based on new observations, there may be previous diagnoses which become inconsistent with the observations and the system description. The set of diagnoses that are still possible is denoted by R and called the set of *remaining* diagnoses; it can be partitioned into two disjoint subsets: (*i*) the set of diagnoses that imply the observations, called the set of *selected* diagnoses and denoted by S, and (*ii*) the set of diagnoses that neither predict nor contradict the observations, called the set of *uncommitted* diagnoses, denoted by U. By definition, $R = S \cup U$ and $S \cap U = \emptyset$.

The posterior probability of a set of behaviour assumptions that is either inconsistent (not in R), a selected diagnosis (in S), or an uncommitted diagnosis (in U) is computed as follows:

$$P(\Delta_C \mid \text{OBS}) = \begin{cases} 0 & \text{if } \Delta_C \notin R \\ \frac{P(\Delta_C)}{P(\text{OBS})} & \text{if } \Delta_C \in S \\ \frac{P(\Delta_C)/m}{P(\text{OBS})} & \text{if } \Delta_C \in U \end{cases} \quad (1)$$

where $m = 1/P(\text{OBS} \mid \Delta_C)$.

Finally, the probability $P(\text{OBS})$ is computed as follows:

$$\begin{aligned} P(\text{OBS}) &= \sum_{\Delta_C \in R} P(\text{OBS}, \Delta_C) \\ &= \sum_{\Delta_C \in S} P(\text{OBS}, \Delta_C) + \sum_{\Delta_C \in U} P(\text{OBS}, \Delta_C) \\ &= \sum_{\Delta_C \in S} P(\Delta_C) + \sum_{\Delta_C \in U} \frac{P(\Delta_C)}{m}. \end{aligned} \quad (2)$$

Computation of $P(\Delta_C)$ is made easy in GDE by assuming independence between components behaving normally or abnormally.

One of the consequences of this assumption is the following proposition.

Proposition 1 *Let $\mathcal{P}_L = (\text{SD}, \text{OBS})$ be a logical diagnostic system with associated joint probability distribution P as defined above for GDE, such that $P(A_c) \ll P(\neg A_c)$ for each $c \in \text{COMPS}$, and let Δ_C and $\Delta_{C'}$ be two consistency-based diagnoses that are both in either S or U, then it holds that:*

$$P(\Delta_C \mid \text{OBS}) \geq P(\Delta_{C'} \mid \text{OBS}) \quad \text{if } C \subseteq C'.$$

Proof. The result follows from the assumption of independence together with $P(A_c) \ll P(\neg A_c)$:

$$\begin{aligned} P(\Delta_C) &= \prod_{c \in C} P(A_c) \prod_{c \in \text{COMPS}-C} P(\neg A_c) \\ &\geq \prod_{c \in C'} P(A_c) \prod_{c \in \text{COMPS}-C'} P(\neg A_c) = P(\Delta_{C'}) \end{aligned}$$

Filling this result into Equation (1) gives the requested outcome. □

For further detail of GDE the reader is referred to the paper by De Kleer and Williams [2]. The following example illustrates how GDE works.

Table 1. Comparison of the values of the diagnostic conflict measure and GDE for the full-adder circuit with observations OBS $= \omega = \{i_1, \bar{i}_2, i_3, o_1, \bar{o}_2\}$ and the probability distribution P, assuming that $P(a_c) = P(o_c \mid a_c) = 0.001$.

k	$X2$	$R1$	$X1$	$A1$	$A2$	conf$[P^{\delta_k}](\omega)$	GDE's $P(\Delta_k \mid \text{OBS})$
1	1	1	1	1	1	−	−
2	1	1	1	1	0	−	−
3	1	1	1	0	1	−	−
4	1	1	1	0	0	−	−
5	1	1	0	1	1	−0.4255	0.99402
6	1	1	0	1	0	−0.4255	$9.9502 \cdot 10^{-4}$
7	1	1	0	0	1	−0.3006	$9.9502 \cdot 10^{-4}$
8	1	1	0	0	0	−0.3006	$9.9601 \cdot 10^{-7}$
9	1	0	1	1	1	−	−
10	1	0	1	1	0	−	−
11	1	0	1	0	1	−	−
12	1	0	1	0	0	−	−
13	1	0	0	1	1	−0.3006	$9.9502 \cdot 10^{-4}$
14	1	0	0	1	0	−0.3006	$9.9601 \cdot 10^{-7}$
15	1	0	0	0	1	−0.3006	$9.9601 \cdot 10^{-7}$
16	1	0	0	0	0	−0.3006	$9.9701 \cdot 10^{-10}$
17	0	1	1	1	1	−	−
18	0	1	1	1	0	−0.1249	$9.9502 \cdot 10^{-4}$
19	0	1	1	0	1	−	−
20	0	1	1	0	0	0	$9.9502 \cdot 10^{-7}$
21	0	1	0	1	1	−0.1247	$9.9502 \cdot 10^{-4}$
22	0	1	0	1	0	−0.1249	$9.9601 \cdot 10^{-7}$
23	0	1	0	0	1	0.0002	$9.9601 \cdot 10^{-7}$
24	0	1	0	0	0	0	$9.9701 \cdot 10^{-10}$
25	0	0	1	1	1	0	$9.9502 \cdot 10^{-4}$
26	0	0	1	1	0	0	$9.9601 \cdot 10^{-7}$
27	0	0	1	0	1	0	$9.9601 \cdot 10^{-7}$
28	0	0	1	0	0	0	$9.9701 \cdot 10^{-10}$
29	0	0	0	1	1	0	$9.9601 \cdot 10^{-7}$
30	0	0	0	1	0	0	$9.9701 \cdot 10^{-10}$
31	0	0	0	0	1	0	$9.9701 \cdot 10^{-10}$
32	0	0	0	0	0	0	$9.9801 \cdot 10^{-13}$

EXAMPLE 2 Reconsider the full-adder shown in Figure 1, where

each component can only be normal or abnormal. Assume that the probability of faulty behaviour of a component is equal to $P(A_c) = 0.001$. Without any observations, the diagnosis space consists of $2^5 = 32$ members, where the diagnosis $\Delta_\emptyset = \{\neg A_c \mid c \in \text{COMPS}\}$ is the most probable diagnosis with probability $P(\Delta_\emptyset) = (1 - P(A_c))^5 = (0.999)^5 \approx 0.995$. When more components are assumed to be faulty, the probabilities decrease quickly to very small values.

Now, suppose that OBS $= \{i_1, \bar{i}_2, i_3, o_1, \bar{o}_2\}$. The new probabilities obtained from GDE are shown in the right-most column of Table 1, where '1' for a component means normal behaviour and '0' means abnormal behaviour. The diagnoses Δ_k, for $k = 1, 3, 4, 9, \ldots, 12, 17, 19$, respectively, are eliminated by these observations. Furthermore, since there are no diagnoses in the set R that imply the two output observations, the set of S is empty and, thus, the set of uncommitted diagnoses U is equal to R. Then, the posterior probability of a diagnosis Δ_k can be computed as follows:

$$P(\Delta_k \mid \text{OBS}) = \frac{P(\Delta_k)/m}{(\sum_{\Delta_C \in U} P(\Delta_C))/m} = \frac{P(\Delta_k)}{\sum_{\Delta_C \in U} P(\Delta_C)},$$

where here $\sum_{\Delta_C \in U} P(\Delta_C) \approx 1.002 \cdot 10^{-3}$. □

In the example, the probability of the Δ_k's that still can be diagnoses become about 1000 times more likely when conditioning on the observations than without observations. However, either with or without observations, the diagnosis with the fewest number of abnormality assumptions is the most likely one. Thus the resulting diagnostic reasoning behaviour is very similar to that obtained by exploiting the concept of subset-minimal diagnosis.

2.3 Bayesian Networks and the Conflict Measure

Let $P(X)$ be a joint probability distribution of the set of discrete binary random variables X. A single random variable taking the values 'true' or 'false' is written as (upright) y and $\bar{y}$, respectively. If we refer to arbitrary values of a set of variables X, sometimes a single variable, this will be denoted by (italic) x. Let $U, W, Z \subseteq X$ be disjoint sets of random variables, then U is said to be *conditionally independent* of W given Z, if for each value u, w and z:

$$P(u \mid w, z) = P(u \mid z), \text{ with } P(w, z) > 0. \tag{3}$$

A *Bayesian network* $\mathcal{B}$ is defined as a pair $\mathcal{B} = (G, P)$, where $G = (V, E)$ is an acyclic directed graph, with set of vertices V and set of arcs E, P is the associated joint probability distribution of the set of random variables X which is associated 1–1 with V. We will normally use the same names for variables and their associated vertices. The factorisation of P respects the independence structure of G as follows: $P(x) = \prod_{y \in x} P(y \mid \pi(y))$, where $\pi(y)$ denotes the values of the parent set of vertex Y. Finally, we will frequently make use of marginalising out particular variables W written as $P(u) = \sum_w P(u, w)$.

Bayesian networks specify probabilistic patterns that must be fulfilled by observations. Observations are random variables that obtain a value through an intervention, such as a diagnostic test. The set of *observations* is denoted by ω. The *conflict measure* has been proposed as a tool for the detection of potential conflicts between observations and a given Bayesian network and is defined as [5]:

$$\text{conf}(\omega) = \log \frac{P(\omega_1)P(\omega_2) \cdots P(\omega_m)}{P(\omega)}, \tag{4}$$

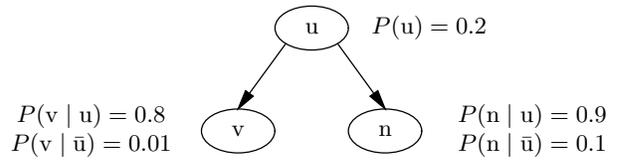with $\omega = \omega_1 \cup \omega_2 \cup \cdots \cup \omega_m$.

Figure 2. Example of a Bayesian network.

The interpretation of the conflict measure is as follows. A zero or negative conflict measure means that the denominator is equally likely or more likely than the numerator. This is interpreted as that the joint occurrence of the observations is in accordance with the probabilistic patterns in P. A positive conflict measure, however, implies negative correlation between the observations and P indicating that the observations do not match P very well.

EXAMPLE 3 Consider the Bayesian network shown in Figure 2, which describes that stomach ulcer (u) may give rise to both vomiting (v) and nausea (n).

Now, suppose that a patient comes in with the symptoms of vomiting and nausea. The conflict measure then has the following value:

$$\text{conf}(\{v, n\}) = \log \frac{P(v)P(n)}{P(v, n)} = \log \frac{0.168 \cdot 0.26}{0.1448} \approx -0.5.$$

As the conflict measure assumes a negative value, there is no conflict between the two observations. This is consistent with medical knowledge, as we do expect that a patient with stomach ulcer displays symptoms of both vomiting and nausea.

As a second example, suppose that a patient has only symptoms of vomiting. The conflict measure now obtains the following value:

$$\text{conf}(\{v, \bar{n}\}) = \log \frac{0.168 \cdot 0.74}{0.0232} \approx \log 5.36 \approx 0.7.$$

As the conflict measure is positive, there is a conflict between the two observations, which is in accordance to medical expectations. □

2.4 Bayesian Diagnostic Problems

A *Bayesian diagnostic system* is denoted as a pair $\mathcal{S}_B = (G, P)$, where P is a joint probability distribution of the vertices of G, interpreted as random variables, and G is obtained by mapping a logical diagnostic system $\mathcal{S}_L = (\text{SD}, \text{COMPS})$ to a Bayesian diagnostic system $\mathcal{S}_B$ as follows [6]:

1. component c is represented by its *input* I_c and *output* O_c vertices, where inputs are connected by an arc to the output;
2. to each component c there belongs an *abnormality vertex* A_c which has an arc pointing to the output O_c.

Figure 3 shows the Bayesian diagnostic system corresponding to the logical diagnostic system shown in Figure 1.

Let O denote the set of all output variables and I the set of all input variables, let o and i denote (arbitrary) values of the set of output and input variables, respectively, and let

$$\delta_C = \{a_c \mid c \in C\} \cup \{\bar{a}_c \mid c \in \text{COMPS} - C\}$$

be the set of values of the abnormality variables A_c, with $c \in$ COMPS. The latter definition establishes a link between Δ_C in logical diagnostic systems and the abnormality variables in Bayesian diagnostic systems.
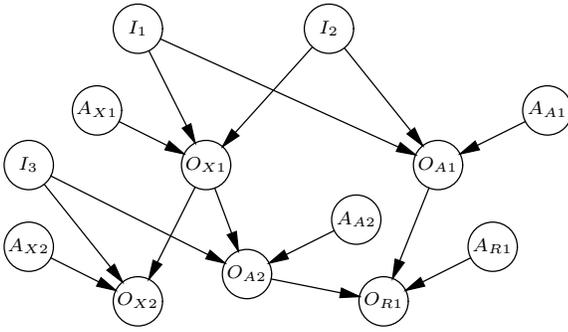
Figure 3. The graphical representation of a Bayesian diagnostic system corresponding to the full-adder in Figure 1.

Due to the independences that hold for a Bayesian diagnostic system, it is possible to simplify the computation of the joint probability distribution P by exploiting the following properties:

Property 1: the joint probability distribution of a set of output variables O can be factorised as follows:

$$P(o) = \sum_{i,\delta_c} P(i,\delta_c) \prod_{c \in \text{COMPS}} P(o_c \mid \pi(o_c)) ; \qquad (5)$$

Property 2: the input variables and abnormality variables are mutually independent of each other, formally: $P(i,\delta_c) = P(i)P(\delta_c)$.

Recall that logical diagnostic problems are logical diagnostic systems augmented with observations; Bayesian diagnostic problems are defined similarly. The input and output variables that have been observed are now referred to as I_ω and O_ω, respectively. The unobserved input and output variables will be referred to as I_u and O_u respectively. The set of actual observations is then denoted by $\omega = i_\omega \cup o_\omega$. Thus, a *Bayesian diagnostic problem* $\mathcal{P}_B = (\mathcal{S}_B, \omega)$ consists of (*i*) a Bayesian diagnostic system representing the components, their behaviour and interaction, and (*ii*) a set of observations ω [4].

In Bayesian diagnostic problems, the normal behaviour of component c is expressed in a probabilistic setting by the assumption that a normally functioning component yields an output value with probability of either 0 or 1. Thus,

$$P(o_c \mid \pi(o_c)) \in \{0, 1\},$$

when the abnormality variable $A_c \in \pi(O_c)$ takes the value 'false', i.e. is $\bar{a}_c$. For the abnormal behaviour of a component c it is assumed that the random variable O_c is conditionally independent of its parent set $\pi(O_c)$ if component c is assumed to function abnormally, i.e. A_c takes the value 'true', written as:

$$P(o_c \mid \pi(o_c)) = P(o_c \mid a_c).$$

Thus, the fault behaviour of an abnormal component cannot be influenced by its environment. We use the abbreviation $P(o_c \mid a_c) = p_c$. Note that this assumption is *not* made when a component is behaving normally, i.e. when $\bar{a}_c$ holds.

3 CONFLICT-BASED DIAGNOSIS

There exists a 1–1 correspondence between a *consistency-based diagnosis* Δ_C of a logical diagnostic problem $\mathcal{P}_L$ and a δ_C for which it holds that $P(\omega \mid \delta_C) \neq 0$ if $\mathcal{P}_B$ is the result of the mapping described above, applied to $\mathcal{P}_L$. The basic idea behind conflict-based

diagnosis is that the conflict measure can be used to rank these consistency-based diagnoses (cf. [4]). We start with the definition of the diagnostic conflict measure.

Definition 1 (*diagnostic conflict measure*) Let $\mathcal{P}_B = (\mathcal{S}_B, \omega)$ be a Bayesian diagnostic problem. The diagnostic conflict measure, denoted by $\text{conf}[P^{\delta_C}](\cdot, \cdot)$, is defined for $P(\omega \mid \delta_C) \neq 0$, as:

$$\text{conf}[P^{\delta_C}](i_\omega, o_\omega) = \log \frac{P(i_\omega \mid \delta_C)P(o_\omega \mid \delta_C)}{P(i_\omega, o_\omega \mid \delta_C)}, \qquad (6)$$

with observations $\omega = i_\omega \cup o_\omega$.

Using the independence properties of Bayesian diagnostic problems we obtain [4]:

$$\text{conf}[P^{\delta_C}](i_\omega, o_\omega) = \log \frac{\sum_i P(i) \sum_{o_u} \prod_c P(o_c \mid \pi(o_c))}{\sum_{i_u} P(i_u) \sum_{o_u} \prod_c P(o_c \mid \pi(o_c))}.$$

where $\pi(O_c)$ may include input variables from I.

The diagnostic conflict measure can take positive, zero and negative values having different diagnostic meaning. Note that the numerator of the diagnostic conflict measure is defined as the probability of the individual occurrence of the inputs and outputs, whereas the denominator is defined as the probability of the joint occurrence of the observations. Intuitively, if the probability of the individual occurrence of the observations is higher than that of the joint occurrence, then the observations do not support each other. Thus, more conflict between diagnosis and observations yields higher (more positive) values of the diagnostic conflict measure. This means that the *sign* of the diagnostic conflict measure, negative, zero or positive, can already be used to rank diagnoses in a qualitative fashion.

This interpretation gives rise to the following definition.

Definition 2 (*(minimal) conflict-based diagnosis*) Let $\mathcal{P}_B = (\mathcal{S}_B, \omega)$ be a Bayesian diagnostic problem and let δ_C be a consistency-based diagnosis of $\mathcal{P}_B$ (i.e. $P(\omega \mid \delta_C) \neq 0$). Then, δ_C is called a conflict-based diagnosis if $\text{conf}[P^{\delta_C}](\omega) \leq 0$. A conflict-based diagnosis δ_C is called minimal, if for each conflict-based diagnosis $\delta_{C'}$ it holds that $\text{conf}[P^{\delta_C}](\omega) \leq \text{conf}[P^{\delta_{C'}}](\omega)$.

In general, the diagnostic conflict measure has the important property that its value can be seen as the overall result of a local analysis of component behaviours under particular logical and probabilistic normality and abnormality assumptions. A smaller value of the diagnostic conflict measure is due to a higher likelihood of dependence between observations, and this indicates a better fit between observations and component behaviours. Consider the following example.

EXAMPLE 4 Reconsider the full-adder circuit example from Figure 1. Let as before $\omega = \{i_1, \bar{i}_2, i_3, o_1, \bar{o}_2\}$. The diagnostic conflict measures for all the possible diagnoses are listed in Table 1.

As an example, the diagnostic conflict measures for the diagnoses $\delta_5, \delta_6, \delta_7$ and δ_8 are compared to one another for the case that the probability $p_{X1} = P(o_{X1} \mid a_{X1}) = 0.001$ and it is explained what it means that, according to Table 1, $\text{conf}[P^{\delta_5}](\omega) = \text{conf}[P^{\delta_6}](\omega) < \text{conf}[P^{\delta_7}](\omega) = \text{conf}[P^{\delta_8}](\omega)$.

First, the diagnoses δ_k, for $k = 6$ and $k = 7$, will be considered in more detail in order to explain the meaning of the diagnostic conflict measure. The difference in value of the diagnostic conflict measure for these two diagnoses can be explained by noting that for δ_6 it is assumed that the adder A1 functions normally and A2 abnormally, whereas for δ_7 it is the other way around. The diagnostic conflict measure of the diagnosis δ_6 is higher than that for δ_7, because if A1

functions normally, then its output has to be equal to 0, whereas if A2 functions normally, then its output has to be equal to 1. Note that it has been observed for R1 that the output is equal to 0. Because 0 is the output of the OR gate R1, its inputs must be 0; therefore, the assumption that A1 functions normally with output 0 offers a better explanation for the output 0 of the R1 gate than the assumption in δ_7 that A2 functions normally (which yields output value 1). Furthermore, since in both diagnoses δ_6 and δ_7 component X1 is assumed to be faulty, and the output of the X1 acts as the input of A2, the assumption about the output of A2 is already relaxed. This also explains the preference of diagnosis δ_6 above δ_7 and why δ_6 is ranked higher than δ_7.

Next, the diagnoses $\delta_7, \delta_8, \delta_{13}, \delta_{14}, \delta_{15}$, and δ_{16} are compared to one another and we explain why it is reasonable that these diagnoses have the same diagnostic conflict measure value (-0.3006). Note that both diagnoses δ_7 and δ_8 include faulty assumption a_{X1} and a_{A1}, and $\delta_{13}, \delta_{14}, \delta_{15}$ and δ_{16} include the faulty behaviours a_{R1} and a_{X1}. Note that for both $\{a_{X1}, a_{A1}\}$ and $\{a_{R1}, a_{X1}\}$, one input of the X2 and the two inputs of R1 are relaxed. Therefore, they yield the same qualitative information about fault behaviour of the system. Below, these results are compared with those by GDE. □

The example above illustrates that comparing the value of the diagnostic conflict measure for different diagnoses gives considerable insight into the behavioural abnormality of a system.

4 COMPARISON

In this section, the diagnostic conflict measure and GDE's probabilistic method are compared to each other in terms of the difference in ranking they give rise to. To start, the main differences between the diagnostic conflict measure and GDE are summarised, which is followed by an example. The example is used to illustrate that the diagnostic conflict measure yields a ranking that, for the probability distribution defined earlier, conveys more useful diagnostic information than the ranking by GDE.

The following facts summarise the differences and similarities between the diagnostic conflict measure and GDE:

1. an abnormality assumption Δ_C is a diagnosis according to GDE iff its associated diagnostic conflict measure is defined, i.e. [4]

$$P(\omega \mid \delta_C) \neq 0 \Leftrightarrow \text{SD} \cup \Delta_C \cup \text{OBS} \nvDash \bot.$$

2. computation of the diagnostic conflict measure requires the conditional probability $p_c = P(o_c \mid a_c)$, i.e. the probability that the component's output is o_c when the component is faulty, this probability is assumed to be always 0 or 1 by GDE.
3. in GDE the probability $P(a_c)$, i.e. the probability that component c functions abnormally, acts as the basis for ranking diagnoses; this probability is not needed to rank diagnoses using conflict-based diagnosis, because it is summed out in the computation of the diagnostic conflict measure.
4. the ranking of a conflict-based diagnosis is based on a local analysis of interactions between inputs and outputs of components, taking into account the probability of particular faulty behaviours of components, and thus can be interpreted as a measure of how well the diagnosis, observations and system behaviour match; GDE offers nothing that is to some extent similar.
5. in GDE assuming more components to be functioning abnormally renders a diagnosis less likely, as proved in Proposition 1; a similar property does not hold for conflict-based diagnosis using the diagnostic conflict measure.

All properties above have already been discussed extensively. Therefore, only the last issue is illustrated.

EXAMPLE 5 Consider the Bayesian diagnostic problem discussed above. Table 1 summarises the results of GDE and conflict-based diagnosis, which makes it easier to compare the results. Note that $\delta_k \equiv \Delta_k$ and $\omega \equiv \text{OBS}$.

Consider again the Bayesian diagnostic problem $\mathcal{P}_B$ with set of observations $\omega = \{i_1, \bar{i}_2, i_3, o_1, \bar{o}_2\}$ and the two diagnoses $\delta_5 = \delta_{\{X1\}} = \{\bar{a}_{X2}, \bar{a}_{R1}, a_{X1}, \bar{a}_{A1}, \bar{a}_{A2}\}$ and $\delta_6 = \delta_{\{X1,A2\}} = \{\bar{a}_{X2}, \bar{a}_{R1}, a_{X1}, \bar{a}_{A1}, a_{A2}\}$.

According to Table 1 the posterior probabilities computed by GDE are equal to $P(\Delta_5 \mid \text{OBS}) = 0.99402$ and $P(\Delta_6 \mid \text{OBS}) = 9.9502 \cdot 10^{-4}$. Thus, Δ_5 is much more likely than Δ_6, which is due to the inclusion of an extra abnormality assumption in Δ_6 in comparison to Δ_5. Consequently, the ranking obtained is compatible with subset-minimality. However, using the diagnostic conflict measure gives, according to Table 1, for both diagnoses the value of -0.4255. This means that relaxing one extra logical and probabilistic constraint, i.e. $A2$ in addition to $X1$, has no effect on the likelihood of the diagnosis in this case.

Next consider the diagnoses Δ_7 and Δ_6, which both have the same number of components assumed to be abnormal, and thus obtain the same ranking according to GDE. However, δ_6 and δ_7 have a different diagnostic conflict measure, as explained in Example 4. □

This example again illustrates that GDE and conflict-based diagnosis rank diagnoses differently. Conflict-based diagnosis really looks into the system behaviour and, based on a local analysis of strength of the various constraints, comes up with a ranking.

5 CONCLUSION AND FUTURE WORK

Conflict-based diagnosis is a new concept in the area of model-based diagnosis that has been introduced recently [4]. In this paper, we have compared this new method with the well-known probabilistic method employed in GDE. It was shown that the probabilistic method underlying conflict-based diagnosis yields detailed insight into the behaviour of a system. As the obtained information differs from information obtained from GDE, it may be useful as an alternative or complementary method.

In the near future, we intend to implement the method as part of a diagnostic reasoning engine in order to build up experience with regard to the practical usefulness of the method.

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