Collision-free Tracking Control of Unicycle Mobile Robots

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***Abstract*—We propose a tracking control with collision avoidance for a group of unicycle mobile robots. A supervisory system assigns to each robot its reference path, together with the desired velocity profile as a function of the position along the path. The robot paths and velocity profiles do not necessarily prevent collisions of the robots. Based on the actual robot coordinates, reference velocities of individual robots are coordinated in real-time such as to ensure collision-free robot movements along the assigned paths. The robot motion controllers realize globally asymptotic stable tracking of the reference trajectories under constraints on the actuator inputs. The controllers proposed here offer several advantages with respect to those in the literature, such as better suppression of tracking errors due to more freedom in controller tuning and relaxed constraints on the reference velocities. The suggested tracking control with collision avoidance has successfully been validated in experiments. It can be used for the realization of autonomous transportation tasks in warehouses, harbors, factories, etc.**

I. INTRODUCTION

GROUP control of robots is employed to realize tasks that are spatially distributed or dangerous, when a single robot cannot accomplish the task or when a different number of robots is needed as the operation proceeds over time, in order to increase robustness in task execution (e.g. by means of redundant robots), to gain flexibility by assigning a multitude of objectives to the robots, etc. Motion coordination is a popular topic of study in multi-robot systems [1]. A broad overview of the research results in group coordination and cooperative control is given in [2].

The transport of goods in warehouses is typically done by means of conveyers. In the absence of redundant conveyer lines, failure in only one conveyer can disable a great deal of the transport in the complete warehouse. Instead of including redundant conveyers, which can be expensive, robustness of the transportation can be improved by means of mobile robots. If a robot fails and obstructs a part of the transportation system, the other ones can dynamically alter their paths to avoid the obstacle. If many robots are used at the same time, their operation should be coordinated such as to efficiently accomplish given tasks without collisions.

The planning and scheduling of the tasks can be done centrally, but in a more flexible approach the robots could negotiate with each other and with the supervisor to achieve robustness to different types of failures. Such a high-level multi-agent coordination can be solved using a holonic approach [3]. Holons [4] are autonomous controllers that cooperate in order to achieve a common goal.

The motion coordination of mobile robots is typically solved by proper path planning, traffic control, formation generation, and formation keeping [1]. In the latter two approaches particular attention is given to the stabilization and tracking control of wheeled mobile robots, since these robots belong to the class of nonholonomic dynamical systems that do not satisfy Brockett's necessary condition for smooth stabilization [5] and no smooth time-invariant stabilizing control law exists for these systems. A great deal of the research aims at developing suitable time-varying stabilizing and tracking controllers for general chained-form nonholonomic systems, including mobile robots, see for instance [6-9] and the references therein. With the exception of [8,9] and some references therein, the stabilization and tracking problems with saturation constraints on control signals have rarely been addressed in the literature, although such constraints are often encountered in practice degrading both stability and control performance.

In this paper we propose a collision-free tracking control strategy for a group of unicycle mobile robots. A supervisory system assigns to each robot its reference path, together with the desired velocity profile as a function of the position along this path. The robot paths and velocity profiles do not necessarily prevent collisions of the robots. Since collisions are not allowed, the reference velocities of individual robots are coordinated in real-time based on the actual robot coordinates, such as to ensure collision-free robot movements. Feedback controllers of the individual robots realize globally asymptotic stable tracking of the reference trajectories under constraints on the actuator inputs.

The main contributions of the paper are: *i*) explicitly accounting for geometric properties of the tracking error dynamics, which facilitates Lyapunov based feedback control design, *ii*) guaranteed global asymptotic tracking of the reference trajectories under less conservative requirements on these trajectories and saturated feedback control, *iii*) high flexibility in controller tuning which allows better suppression of the tracking errors and higher robustness against disturbances and parasitic dynamics, *iv*) formulation of an effective method for collision-free robot coordination, and *v*) verification of this method in realistic experiments.

In section 2 we present background mathematical models and tools. In Section 3 we solve the global trajectory tracking problem. In Section 4 we propose a method for collision-free robot coordination. Experimental results are presented in Section 5. Conclusions are given at the end.

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II. Mathematical Preliminaries

Here, we recall some concepts from control systems theory that will be used in the subsequent sections.

A. Robot Kinematic Model and Tracking Error Dynamics

We consider a unicycle mobile robot, depicted in Fig. 1, whose configuration is described by the following well-known kinematic model (see [6-9] and references therein):

$$\begin{aligned} \dot{x} &= v\cos\theta, \\ \dot{y} &= v\sin\theta, \\ \dot{\theta} &= \omega, \end{aligned} \tag{1}$$

where v and ω are the forward and steering velocities, respectively, x and y are the planar coordinates of the robot midpoint O_e in the world coordinate frame Oxy, and θ is the heading angle relative to the x-axis of the world frame.

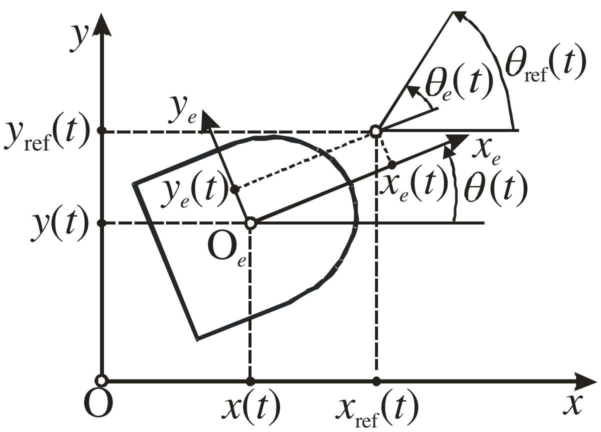

Fig. 1. Configuration and error coordinates of a unicycle mobile robot.

In this paper, we consider the tracking problem: we aim to design control laws for v and ω such that the robot asymptotically tracks a reference trajectory specified as

$$\mathbf{p}_{\text{ref}}(t) = \begin{bmatrix} x_{\text{ref}}(t) \\ y_{\text{ref}}(t) \\ \theta_{\text{ref}}(t) \end{bmatrix}, \tag{2}$$

where x_{ref}, y_{ref}, and θ_{ref} are all defined in the world frame. For control design, we consider tracking errors represented in the coordinate frame of the robot $O_e x_e y_e$ (see, e.g., [6-8]):

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\text{ref}} - x \\ y_{\text{ref}} - y \\ \theta_{\text{ref}} - \theta \end{bmatrix}. \tag{3}$$

Dynamics of the errors (x_e, y_e, θ_e) have been modeled in [6]:

$$\dot{\mathbf{e}}_{xy}(t) = -\omega(t)\mathbf{S}\mathbf{e}_{xy}(t) + \begin{bmatrix} v_{\text{ref}}(t)\cos\theta_e(t) - v(t) \\ v_{\text{ref}}(t)\sin\theta_e(t) \end{bmatrix},$$

$$\dot{\theta}_e(t) = \omega_{\text{ref}}(t) - \omega(t), \tag{4a}$$

$$\mathbf{e}_{xy} = \begin{bmatrix} x_e \\ y_e \end{bmatrix}, \quad \mathbf{S} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{4b}$$

Here, $v_{\text{ref}}(\text{t}) = \sqrt{(\dot{x}_{\text{ref}}(t))^2 + (\dot{y}_{\text{ref}}(t))^2}$, $\omega_{\text{ref}}(t) = \dot{\theta}_{\text{ref}}(t)$. The matrix $\mathbf{S}$ is skew-symmetric, featuring the well-known property (5) which is very useful for control design:

$$\mathbf{x}^T\mathbf{S}\mathbf{x} \equiv 0, \ \forall \mathbf{x} \in \mathrm{R}^2. \tag{5}$$

B. A Concept from Stability Theory

Lemma 1. Consider a scalar system

$$\dot{x}(t) = -\phi(x(t)) + p(t), \tag{6}$$

where $\phi(x(t))$ and $p(t)$ are bounded and uniformly continuous functions such that $x\phi(x) > 0$ if $x \neq 0$ and $\phi(0) = 0$. If, for any $t_0 \geq 0$ and any initial condition $x(t_0)$, the solution $x(t)$ is bounded and $\lim_{t\to\infty} x(t) = 0$, then

$$\lim_{t\to\infty} p(t) = 0. \tag{7}$$

Proof: We use standard epsilon-delta argumentation. For any ϵ, we can find $t(\epsilon)$ such that $|x(\tau)| < \epsilon$ and $|\phi(x(\tau))| < \epsilon$ for all $\tau \geq t$. Then for any $\delta \in \mathrm{R}_+$, we determine from (6):

$$\begin{aligned} \int_t^{t+\delta} p(\tau)\mathrm{d}\tau &\leq \int_t^{t+\delta} |p(\tau)|\mathrm{d}\tau \\ &\leq \left|\int_t^{t+\delta} \mathrm{d}x(\tau)\right| + \int_t^{t+\delta} |\phi(\tau)|\mathrm{d}\tau \\ &< |x(t+\delta)| + |x(t)| + \epsilon \int_t^{t+\delta} \mathrm{d}\tau \\ &< \epsilon(2+\delta). \end{aligned}$$

Since ϵ can be made arbitrarily small, then $|p(\tau)|$ is also arbitrarily small for $\tau \geq t(\epsilon)$. ■

C. Saturation and Penalty Functions

First, we introduce saturation functions in a similar way as in [8]. We will use these functions in the control design presented in Section 3. We define a set $\mathrm{BF}_{r,k}$ of uniformly continuous and bounded functions indexed by (possibly time-variant) bounded parameters $r, k \in \mathrm{R}_+$:

$$\mathrm{BF}_{r,k} = \{\phi_{r,k}: \mathrm{R} \to \mathrm{R} \mid \phi_{r,k} \text{ is uniformly continuous and } -r \leq \phi_{r,k}(kx) \leq r \ \ \forall x \in \mathrm{R}\} \tag{8}$$

and a corresponding set of saturation functions $\mathrm{S}_{r,k}$

$$\mathrm{S}_{r,k} = \{\phi_{k,r}: \mathrm{R} \to \mathrm{R} \mid \phi_{r,k} \in \mathrm{BF}_{r,k}, \ s\phi_{r,k}(ks) > 0 \text{ for all } s \neq 0, \phi_{r,k}(0) \equiv 0\}. \tag{9}$$

Examples of nontrivial, yet simple, functions in S_r are:

$$\phi_{r,k}(kx) = r\frac{kx}{\sqrt{1+(kx)^2}}, \tag{10a}$$

$$\phi_{r,k}(kx) = r\tanh(kx). \tag{10b}$$

Unlike [8], we use a time-varying instead of constant design parameter r and introduce an additional parameter k, in order to improve the performance of the resulting controller.

Second, we introduce a set P_γ of continuous, monotone and bounded penalty functions indexed by a constant parameter $\gamma \in \mathrm{R}_+$:

$$\mathrm{P}_\gamma = \{\delta_\gamma: \mathrm{R} \to \mathrm{R} \mid \delta \text{ is continuous, monotone and } \delta_\gamma(x) = 0 \ \ \forall x \in \mathrm{R}_-, 0 \leq \delta_\gamma(x) \leq 1 \text{ if } 0 \leq x \leq \gamma, \ \delta_\gamma(x) = 1 \text{ if } x > \gamma\}. \tag{11}$$

An example of a function in P_γ is

$$\delta_\gamma(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{\gamma}\left(x - \frac{\gamma\sin(2\pi x/\gamma)}{2\pi}\right), & 0 \leq x \leq \gamma \\ 1, & x > \gamma \end{cases} \tag{12}$$

A penalty function is used in our method for collision avoidance presented in Section 4.

III. Design of Tracking Controller with Saturation

In this section we will derive a tracking controller which guarantees global asymptotic stability of the tracking error dynamics (4a,b). Although this derivation will conceptually mimic the approach followed in [7,8], it will result in additional features to be highlighted at the end of this section.

Our objective is to find time-varying state-feedback controllers of the form

$$v = v^*\left(t, \mathbf{e}_{xy}, \theta_e\right),\ \omega = \omega^*\left(t, \mathbf{e}_{xy}, \theta_e\right) \tag{13}$$

such as to ensure global asymptotic stability of the tracking error dynamics (4a,b) under constraints on the inputs:

$$|v(t)| \le v_{\max},\ \ |\omega(t)| \le \omega_{\max},\ \forall t \ge 0, \tag{14}$$

where $v_{\max}$ and $\omega_{\max}$ are given positive constants.

In the following we show that the following control laws solve the given tracking problem:

$$v(t) = v_{\text{ref}}(t)\cos(\theta_e(t)) + \phi_{k_x(t),c_x}(c_x x_e(t)), \tag{15a}$$

$$\omega(t) = \omega_{\text{ref}}(t) + \frac{k_y k_{xy} y_e(t) v_{\text{ref}}(t)}{\sqrt{1+\left(k_{xy}x_e(t)\right)^2+\left(k_{xy}y_e(t)\right)^2}}\frac{\sin(\theta_e(t))}{\theta_e(t)} + \phi_{k_\theta(t),c_\theta}(c_\theta \theta_e(t)), \tag{15b}$$

where $k_x, k_y, k_{xy}, k_\theta, c_x, c_\theta \in \mathrm{R}_+$ are design parameters, and $\phi_{k_x,c_x} \in \mathrm{S}_{k_x,c_x}$, $\phi_{k_\theta,c_\theta} \in \mathrm{S}_{k_\theta,c_\theta}$. Here, the sets S_{k_x,c_x} and $\mathrm{S}_{k_\theta,c_\theta}$ are defined by (9). The parameters k_y, k_{xy}, c_x, and c_θ are constant, while k_x and k_θ can be time-variant. Note that

$$\frac{\sin\theta_e}{\theta_e} = \int_0^1 \cos\,(s\theta_e)\mathrm{d}s \tag{16}$$

is a smooth function in θ_e and recall that

$$\lim_{\theta_e \to 0}\frac{\sin\theta_e}{\theta_e} = 1. \tag{17}$$

We establish the following result.

Theorem 1: Consider the unicycle robot dynamics (1) with inputs v and ω constrained according to (14). Assume that the desired trajectory in (2) is such that the following inequalities hold: $|v_{\text{ref}}(t)| < v_{\max}$, $|\omega_{\text{ref}}(t)| < \omega_{\max}$, $\forall t \ge 0$. Consider the tracking controller (15). We distinguish two cases regarding properties of the reference velocities:

1. Suppose that v_{ref} is nonzero, bounded and uniformly continuous over $t \in [0,\infty)$, while ω_{ref} is bounded over $t \in [0,\infty)$. If parameters in (15) are chosen such that $k_x, k_y, k_{xy}, k_\theta, c_x, c_\theta \in \mathrm{R}_+$ and

   $$k_y,\ k_{xy},\ c_x, \text{ and } c_\theta \text{ are positive constants}, \tag{18a}$$
   $$k_x(t) \le v_{\max} - |v_{\text{ref}}(t)|, \tag{18b}$$
   $$k_\theta(t) \le \omega_{\max} - |\omega_{\text{ref}}(t)| - k_y|v_{\text{ref}}(t)|, \tag{18c}$$

   then $\left(\mathbf{e}_{xy}, \theta_e\right) = \mathbf{0}$ is a globally asymptotically stable equilibrium of the tracking error dynamics (4a,b), i.e. the tracking control problem is solved.
2. Suppose that v_{ref} is bounded, uniformly continuous over $t \in [0,\infty)$, and it converges to zero as $t \to \infty$, while ω_{ref} is nonzero, bounded and uniformly continuous over $t \in [0,\infty)$. With the design parameters chosen as in (18), controller (15) solves the given tracking problem.

Proof: Consider the positive definite and proper Lyapunov function candidate $V\left(\mathbf{e}_{xy}, \theta_e\right)$:

$$V = \frac{k_y}{k_{xy}}\sqrt{1 + \left(k_{xy}\right)^2\left(\mathbf{e}_{xy}\right)^T\mathbf{e}_{xy}} + 0.5(\theta_e)^2 - \frac{k_y}{k_{xy}}, \tag{19}$$

where $k_y, k_{xy} \in \mathrm{R}_+$ are constant design parameters. Differentiating V along the solutions of the closed-loop system (4),(15) yields

$$\frac{\mathrm{d}}{\mathrm{d}t}V\left(\mathbf{e}_{xy}, \theta_e\right) = \frac{k_y k_{xy}\left(\mathbf{e}_{xy}\right)^T\dot{\mathbf{e}}_{xy}}{\sqrt{1+\left(k_{xy}\right)^2\left(\mathbf{e}_{xy}\right)^T\mathbf{e}_{xy}}} + \theta_e\dot{\theta}_e$$

$$= \frac{k_y k_{xy}\left(-\omega\left(\mathbf{e}_{xy}\right)^T\mathbf{S}\mathbf{e}_{xy} + \left(\mathbf{e}_{xy}\right)^T\begin{bmatrix}v_{\text{ref}}\cos\theta_e - v\\ v_{\text{ref}}\sin\theta_e\end{bmatrix}\right)}{\sqrt{1+\left(k_{xy}\right)^2\left(\mathbf{e}_{xy}\right)^T\mathbf{e}_{xy}}} + \theta_e(\omega_{\text{ref}} - \omega)$$

$$= \frac{k_y k_{xy}[x_e \quad y_e]\begin{bmatrix}v_{\text{ref}}\cos\theta_e - v\\ v_{\text{ref}}\sin\theta_e\end{bmatrix}}{\sqrt{1+\left(k_{xy}x_e\right)^2+\left(k_{xy}y_e\right)^2}} + \theta_e(\omega_{\text{ref}} - \omega)$$

$$= -\frac{k_y k_{xy} x_e \phi_{k_x,c_x}(c_x x_e)}{\sqrt{1+\left(k_{xy}x_e\right)^2+\left(k_{xy}y_e\right)^2}} - \theta_e\phi_{k_\theta,c_\theta}(c_\theta\theta_e) \le 0. \tag{20}$$

We use property (5) to obtain the expression given in the third row of (20). The last inequality in (20), which follows from the definition of ϕ_{k_x,c_x} and ϕ_{k_θ,c_θ} in (9), implies that the trajectories of the tracking errors $x_e(t)$, $y_e(t)$, $\theta_e(t)$ are uniformly bounded for $t \in [0,\infty)$. It remains to show global asymptotic convergence of these errors to $\left(\mathbf{e}_{xy}, \theta_e\right) = \mathbf{0}$.

From the last inequality in (20) we can determine

$$0 \ge \int_0^\infty \mathrm{d}V(t) \ge -\int_0^\infty\left[k_y k_{xy} x_e(t)\phi_{k_x(t),c_x}\left(c_x x_e(t)\right) + \theta_e(t)\phi_{k_\theta(t),c_\theta}(c_\theta\theta_e(t))\right]\mathrm{d}t, \tag{21}$$

which implies that the integrals at the right hand side exist and are finite. Consequently, $x_e\phi_{k_x,c_x}$, $\phi_{k_\theta,c_\theta} \in L_1(0,\infty)$. By assumption, $\phi_{k_x(t),c_x}(c_x x_e(t))$ and $\phi_{k_\theta(t),c_\theta}(c_\theta\theta_e(t))$ are uniformly continuous for $t \in [0,\infty)$. Since x_e and θ_e are solutions of the continuous error dynamics (4a,b), which is excited with inputs (15a,b) generated from uniformly continuous functions and bounded $v_{\text{ref}}(t)$ and $\omega_{\text{ref}}(t)$, then $x_e(t)\phi_{k_x(t),c_x}(c_x x_e(t))$ and $\theta_e(t)\phi_{k_\theta(t),c_\theta}(c_\theta\theta_e(t))$ must be uniformly continuous for $t \in [0,\infty)$. By definition (9), $x_e\phi_{k_x,c_x}(c_x x_e)$ and $\theta_e\phi_{k_\theta,c_\theta}(c_\theta\theta_e)$ are always non-negative. With the help of Barbălat's lemma [10] (pp. 211), we get

$$\lim_{t\to\infty}\left[x_e(t)\phi_{k_x(t),c_x}(c_x x_e(t)) + \theta_e(t)\phi_{k_\theta(t),c_\theta}(c_\theta\theta_e(t))\right] = 0, \tag{22}$$

implying

$$\lim_{t\to\infty}[|x_e(t)| + |\theta_e(t)|] = 0. \tag{23}$$

It remains to prove that

$$\lim_{t\to\infty} y_e(t) = 0. \tag{24}$$

First consider the case when $v_{\rm ref}(t)$ remains nonzero. In the closed-loop system (4),(15), the θ_e equation becomes:

$$\dot{\theta}_e(t) = -\phi_{k_\theta(t),c_\theta}(c_\theta\theta_e(t)) - \frac{k_y k_{xy} y_e(t) v_{\rm ref}(t)}{\sqrt{1+(k_{xy}x_e(t))^2+(k_{xy}y_e(t))^2}}\frac{\sin\theta_e(t)}{\theta_e(t)}. \tag{25}$$

A direct application of the Lemma 1 on (25) gives

$$\lim_{t\to\infty}\left(\frac{k_y k_{xy} y_e(t) v_{\rm ref}(t)}{\sqrt{1+(k_{xy}x_e(t))^2+(k_{xy}y_e(t))^2}}\frac{\sin\theta_e(t)}{\theta_e(t)}\right) = 0. \tag{26}$$

Having in mind (17), (23), and that $v_{\rm ref}(t)$ remains nonzero, property (24) must hold. Note that expression (25) does not depend on $\omega_{\rm ref}(t)$. Consequently, global asymptotic stability of the zero equilibrium of the closed-loop system (4),(15) does not require uniform continuity of $\omega_{\rm ref}(t)$.

Now consider the case when $\lim_{t\to\infty} v_{\rm ref}(t) = 0$, while $\omega_{\rm ref}(t)$ remains nonzero. In the closed-loop system (4),(15), the x_e equation becomes:

$$\lim_{t\to\infty}\dot{x}_e(t) = \lim_{t\to\infty}\big[-\phi_{k_x(t),c_x}(c_x x_e(t)) + {} + y_e(t)\big(\omega_{\rm ref}(t) + \phi_{k_\theta(t),c_\theta}(c_\theta\theta_e(t))\big)\big]. \tag{27}$$

A direct application of the Lemma 1 on (27), while using (23), implies that $y_e(t)\omega_{\rm ref}(t)$ must converge to zero as $t\to\infty$, which, in turn, implies (24). Uniform continuity of $\omega_{\rm ref}(t)$ is a precondition for using the Lemma 1 in this case.

Selection of the design parameters according to (18a-c) trivially ensures that the control inputs (15a,b) fulfill (14). ■

Let us point out what are the advantages of our controller. As first, the term $y_e/\sqrt{1+(k_{xy}x_e)^2+(k_{xy}y_e)^2}$ in (15b) ensures that ω monotonously depends on y_e, which implies that ω is very sensitive to large y_e. Consequently, our controller is capable of quick suppression of large y_e-errors. The second advantage is that we make use of the design parameters k_{xy}, c_x, and c_θ to gain high freedom in tuning the transient behavior of the closed-loop system. With these parameters we can speed-up correction of the tracking errors. The third advantage is that we introduce the possibility of using time-varying design parameters $k_x(t)$ and $k_\theta(t)$. We can treat these gains as complements of the reference velocities $v_{\rm ref}(t)$ and $\omega_{\rm ref}(t)$, respectively. In this way, without violating constraints (14), we can allow high-gain feedback during periods of slower movements. Increased feedback gains can improve motion performance at low velocities, especially if a mobile robot is subject to parasitic dynamics and disturbances (e.g., friction on the robot wheels, noise in the feedback signals). The last advantage is that we emphasize that uniform continuity of $\omega_{\rm ref}(t)$ is not needed in case of $v_{\rm ref}(t) \neq 0$. This relaxation is especially relevant for use of mobile robots in distribution centers where, for the sake of time-efficiency, it is desirable that $v_{\rm ref}(t)$ is kept at a maximum value as long as possible on a given robot path. While $v_{\rm ref}(t)$ is kept constant, $\omega_{\rm ref}(t)$ may become discontinuous (yet bounded), especially at blending points between straight- and curved-parts of the robot path. This will be illustrated in a case-study in Section 5.

Compared with the results presented in [8], our controller offers faster suppression of large y_e-errors, more freedom in tuning the transient behavior of the closed-loop system, and better error reduction at lower velocities. It also allows more relaxed requirements on $\omega_{\rm ref}$ than considered in [8].

IV. A Collision-free Robot Coordination Method

To achieve collision-free coordination of a group of n mobile robots, we propose feedback-based adjustments of their reference velocities $v_{{\rm ref},i}(t)$, $i \in \{1,2,\dots,n\}$. The adjustments can be done by means of penalty functions from the set P_γ defined by (11). Within the robot group, we determine mutual distances between the robots. The total number E of distances equals the binomial coefficient:

$$\mathrm{E} = \binom{n}{2} = \frac{n!}{2(n-2)!}. \tag{28}$$

If the distance between two robots is below some predefined threshold, then the desired velocity of the robot of lower priority must be reduced (penalized). We assume that within the group it is always possible to establish unequal priorities among all members, so adjustments of the reference velocities do not lead to deadlocks in robot movements. Unequal priorities can be established by rules that are characteristic to distribution centers: (*i*) one-way street policy, (*ii*) no robot may overtake another during failure-free operation, and (*iii*) on a cross-road, robots coming from the right hand-side should have priority over the others. If the given rules still allow equal priorities to some robots, then shorter distance to a target increases the priority of one robot over another.

Consider first the situation with two mobile robots (1 and 2) only. The distance to collision $\Delta_{1,2}$ between these robots can be determined by subtracting the robot diameter D from the Euclidean distance between the robot midpoints:

$$\Delta_{1,2} = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} - \mathrm{D}. \tag{29}$$

We postulate that if $\Delta_{1,2} < \gamma$, where γ is some threshold, then the desired velocity of a robot of lower priority is penalized. If robot 1 is of lower priority, then it's velocity should be adjusted as:

$$v_{{\rm ref},1}(t) = v_{{\rm des},1}(t)\delta_\gamma(\Delta_{1,2}), \tag{30}$$

where $v_{{\rm des},1}(t)$ is the desired velocity of robot 1, and δ_γ is a penalty function from the set (11). If another robot has lower priority, then its desired velocity should be reduced in the similar way. Note that for robot coordination we make use of the feedback variables (x_1, y_1) and (x_2, y_2).

Now we consider a group of n robots. Without loss of generality, we may assume that the same γ holds for all the robot pairs from the group. If non-equal priorities are assigned to these robots, then $v_{\text{des},i}(t)$, such that $|v_{\text{des},i}(t)| < v_{\max}$ for all $i \in \{1,2,\ldots,n\}$, can be penalized as follows:

$$v_{\text{ref},i}(t) = v_{\text{des},i}(t) \prod_{\substack{j=1 \\ (j \neq i)}}^{n} \delta_{\gamma,i}(\Delta_{i,j}). \tag{31}$$

Here, $\Delta_{i,j}$ is determined in accordance with (29) and $\delta_{\gamma,i}$ is a penalty function from the set (11). Since $\Delta_{i,j} \equiv \Delta_{j,i}$, one needs to determine E distances to collisions, where E is given by (28). For coordination of n robots, we use the feedback variables (x_i, y_i), $i \in \{1,2,\ldots,n\}$.

The proposed robot coordination does not hamper global stability of the zero equilibrium of the closed-loop system (4),(15), since $v_{\text{ref},i}(t)$ remains bounded within $[0, v_{\text{des},i}(t)]$ and uniformly continuous. The latter is ensured by uniform continuity of the penalty functions belonging to the set (11). As long as $v_{\text{ref},i} > 0$, global asymptotic stability is maintained (Theorem 1). If suppression of the desired velocity leads to $v_{\text{ref},i} = 0$, then asymptotic stability is guaranteed if ω_{ref} is nonzero, bounded and uniformly continuous.

V. Experimental Case-study

To illustrate the results presented so far, we conduct experiments with mobile robots operating in a warehouse-like environment. The main components of the experimental setup are: four mobile robots (E-puck [11]), a PC and a camera. The PC generates robot trajectories, uses camera images to determine actual linear and angular coordinates of each robot, and implements algorithms for motion coordination and tracking control. The robots receive control inputs from the PC via a BlueTooth protocol.

Each robot is supposed to move along a given path with the desired linear velocity $v_{\text{des},i}(s_i), i \in \{1,2,3,4\}$. Here, s_i is the distance between the initial and a given point on this path. To achieve an experiment where robot collisions may easily occur in the absence of motion coordination, we start the motion of each robot at a different time-instant and assign mutually different desired velocities to all robots. Ideally, the reference velocity $v_{\text{ref},i}$ at distance s_i coincides with the desired one. The layout of the path of robot i and the corresponding $v_{\text{ref},i}(s_i(t))$ determine the references $x_{\text{ref},i}(t)$, $y_{\text{ref},i}(t)$, and $\theta_{\text{ref},i}(t)$. To verify robustness of the controllers to the initial errors, values $(x_i(0), y_i(0), \theta_i(0))$ are chosen to be different from the reference ones.

We use the following tracking controller:

$$v_i(t) = v_{\text{ref},i}(t)\cos\theta_{e,i}$$

$$+\left(v_{\max} - |v_{\text{ref},i}(t)|\right)\tanh\left(20x_{e,i}(t)\right), \tag{32a}$$

$$\omega_i(t) = \omega_{\text{ref},i}(t) + \frac{11v_{\text{ref},i}(t)20y_{e,i}(t)}{\sqrt{1+\left(20x_{e,i}(t)\right)^2+\left(20y_{e,i}(t)\right)^2}} \frac{\sin(\theta_{e,i}(t))}{\theta_e(t)} +$$

$$+\left(\omega_{\max} - |\omega_{\text{ref},i}(t)| - 11v_{\text{ref},i}(t)\right)\tanh\left(0.5\theta_{e,i}(t)\right). \tag{32b}$$

Here, $v_{\max}$ and $\omega_{\max}$ are the maximum values of the control signals: $v_{\max} = 0.1$ [m/s], $\omega_{\max} = 2$ [rad/s]. Controllers (32a,b) have time-varying feedback gains $v_{\max} - |v_{\text{ref},i}(t)|$ and $\omega_{\max} - |\omega_{\text{ref},i}(t)| - 11v_{\text{ref},i}(t)$. At lower velocities these gains get higher, which improves robustness against disturbances and parasitic dynamics. Time-variations of the feedback gains do not hamper constraints (14), since by design both controllers are saturated. We also use design gains in arguments of the saturation function $\tanh(.)$, in order to gain more freedom in tuning the transient behavior of the closed-loop system. In our experiments, the controller (32) is applied at a sampling rate of 25 [Hz].

In the first experiment, we disable the algorithm for collision-free robot coordination presented in Section 4 and let $v_{\text{ref},i}(s_i) \equiv v_{\text{des},i}(s_i)$, $i \in \{1,2,3,4\}$. Without motion coordination, robots 1 and 2 collide after 27 [s]. This is verified by Fig. 2 showing the actual distances to collisions.

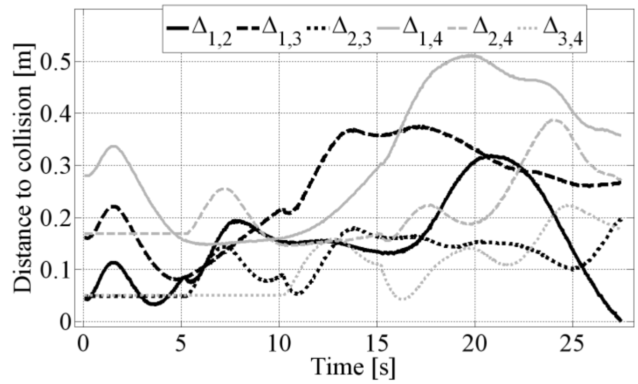

Fig. 2. Distances to collision in the first experiment.

In the second experiment, we adopt the same desired velocities as in the first one, but we do enable coordination among the robots. As a result, all the robots successfully travel along the given reference paths without collisions. In Fig. 3 we show the reference and actual paths in the second experiment. The corresponding plots of distances to collision, shown in Fig. 4, confirm collision avoidance.

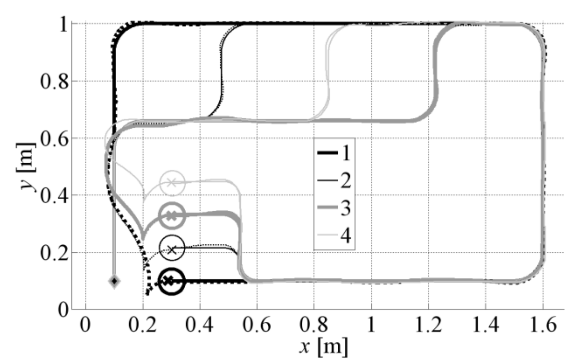

Fig. 3. Reference (solid) and actual (dotted) robot paths in the second experiment; the initial reference and actual positions are indicated with '◊' and '×', respectively; the final actual positions are indicated by circles.

In Fig. 5, we show the desired, reference, and actual linear velocities, while in Fig. 6 we show the reference and actual angular velocities. Note that the actual velocities shown in these figures represent the control signals. Apparently, both the controls meet the constraints (14). We may observe periods where the reference linear velocities are below the desired ones. In these periods the penalty functions are active, i.e., the robots motions are coordinated such as to avoid collisions. Thanks to continuity of the applied penalty functions, the reference linear velocities remain uniformly continuous. We may also notice that the reference angular speeds are not uniformly continuous. In particular, discontinuities occur exactly at the blending points between straight-line and circular segments of the robot paths. Despite these discontinuities, our control design ensures that the asymptotic tracking is achieved with bounded control signal.

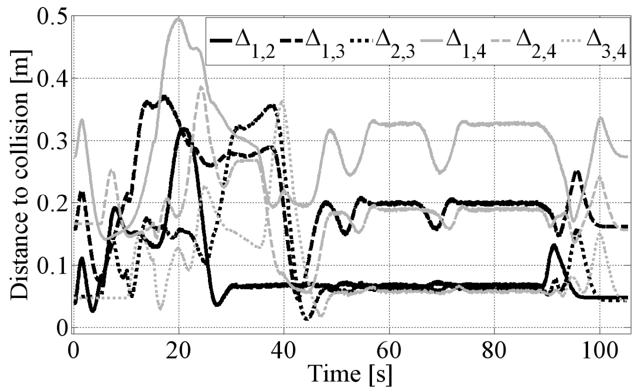

Fig. 4. Distances to collision in the second experiment.

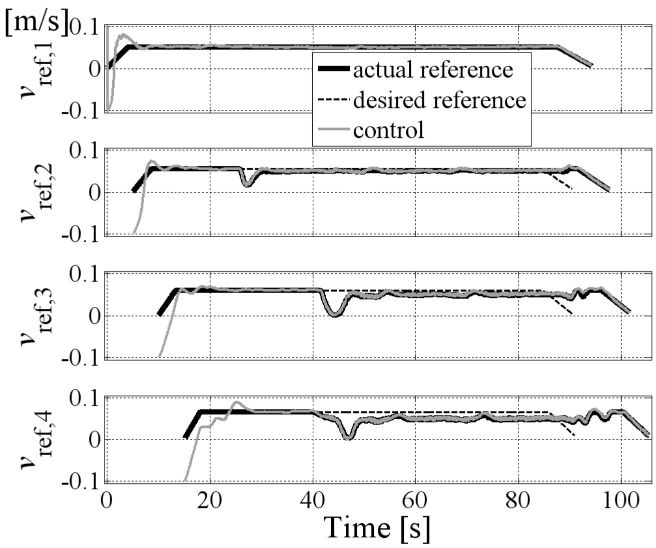

Fig. 5. Linear velocities in the second experiment.

VI. Conclusion

In this paper, we derive and verify a feedback control algorithm for globally stable asymptotic tracking of reference trajectories of unicycle mobile robots under constraints on the robot control signals. We represent a well- known kinematic model of unicycle robots in a form which facilitates derivation of the tracking controller.

The advantages of using our controller are: *i*) fast correction for large position errors in y-direction, *ii*) high freedom in tuning the transient behavior of the closed-loop system, *iii*) use of time-varying feedback gains for better robustness against disturbances and parasitic dynamics, and *iv*) allowing use of discontinuous (yet bounded) reference angular velocity ω_{ref} if the reference linear velocity v_{ref} is nonzero. The last advantage is especially relevant for use of mobile robots in transportation tasks, where, for the sake of time-efficiency, it is desirable to keep v_{ref} at the maximum. While v_{ref} is kept constant, ω_{ref} may become discontinuous, especially at blending points between straight-line and curved segments of the robot path.

We suggest an effective feedback method for collision-free robot coordination. This method penalizes the reference linear velocity v_{ref} of robots of lower priority. To avoid deadlocks in the robot movements, all the robots must have non-equal priorities during execution of their tasks. The robot coordination has been successfully verified in the experiments, confirming realization of the reference robot paths without collisions.

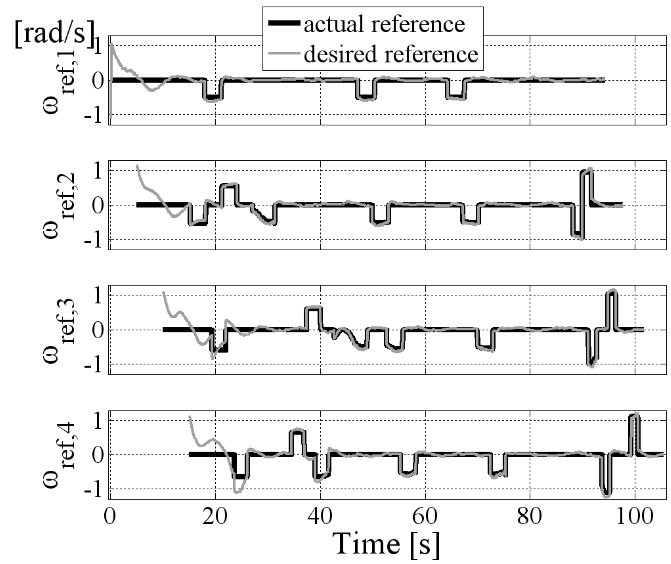

Fig. 6. Angular velocities in the second experiment.

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